Waveform Tomography: theory and practice

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Abstract

Waveform tomography uses the original waveform recordings and wave equation modelling to extract high resolution tomographic images from seismic data. Theoretical and synthetic studies support the conclusion that waveform tomography can potentially image features at sub-wavelength scales.

Introduction

In this talk I will review the theory and practice of *Waveform Tomography* (also referred to as *Full Waveform Inversion* or *Diffraction Tomography*). Many current tomographic methods use the arrival time information, extracted from the original waveform recordings either by manual or automatic, cross-correlation based methods; I refer to these methods generically as *traveltime tomography* methods. Waveform tomography differs from more common methods of tomographic inversion in two major aspects:

- The input data consist of the seismic waveforms themselves (as opposed to traveltimes, amplitudes or some other secondary attribute of the recorded data)
- The underlying numerical method is based on the full wave equation (as opposed to a ray approximation or a Born approximation)

The features above make waveform tomography less approximate and consequently better resolved than traveltime tomography. The same features also make algorithms for waveform tomography more difficult to develop, less robust and more expensive to use than traveltime tomography. As we shall see, it is generally impossible to proceed with waveform tomography without first carrying out traveltime tomography in order to establish a starting model.

In this presentation I will review the fundamental issues of scale that control the resolution of traveltime methods, I will briefly state the physical and numerical principles used in the development of waveform tomography and I will provide some synthetic studies that illustrate both the advantages and potential pitfalls in waveform tomography. I will conclude with a brief review of a number of cross-hole tomography case studies in which waveform tomography has made a demonstrable contribution to the state of geological knowledge of the target.

Scales, wavelengths and Fresnel zones

The characteristic length scale for a seismic wave is its wavelength, λ related to the frequency, f by the fundamental relationship

$$\lambda = \frac{c}{f}, \quad (1)$$

in which c is the propagation velocity of the wave. The general rule is that the high frequencies are lost most rapidly to friction during propagation. Thus, we never seem to have small enough wavelengths: the further the waves travel, the less short wavelength energy there is. Wavelength of course is a critical factor in understanding the spatial resolution of a given method; we generally accept from the outset that the tomographic images we obtain will be limited by the wavelength in some way.

A second, less widely appreciated control on the resolution of tomographic methods is propagation distance. The further a seismic wave travels after encountering a velocity anomaly, the less evident the effect of the anomaly on the wavefronts. Due to diffraction effects, the wavefronts tend to "heal", gradually erasing the effect of the original disturbance. The effect of wavefront healing is that a second scale parameter, the width of the *first Fresnel zone* also plays a role in limiting the resolution. This scale parameter is given by

$$\sqrt{\lambda L}, \qquad (2)$$

where L is the propagation distance the wave travels from source to receiver. The Fresnel zone can be thought of as defining an effective "ray-width", implying that structure far away from the ray path can still influence the wavefronts (e.g., Nolet 1987). The influence of this effect on the resolution of traveltime tomography has been evaluated theoretically by Williamson (1991) and Schuster (1996), and further tested numerically by Williamson and Worthington (1993).

Mathematical physics of tomography

An interesting comparison of traveltime and waveform tomography can be made by considering the following two expressions: for traveltime, ray-based methods we may express the relationship between an anomaly in the slowness field, $\delta s(\mathbf{x})$ and the traveltime anomaly $\delta T(\mathbf{r}, \mathbf{s})$ between a source at $\mathbf{s}$ and a receiver at $\mathbf{r}$ as

$$\delta T(\mathbf{r}, \mathbf{s}) = \int d^3\mathbf{x}\, s(\mathbf{x})\, L_o(\mathbf{x}; \mathbf{r}, \mathbf{s}), \qquad (3)$$

where L_o is a thin, pencil-like path through the media representing the ray path. For waveform tomography the relationship between the anomaly in the squared slowness field and the "scattered wave", $U_{sc}(\mathbf{r}, \mathbf{s}; \omega)$ at a frequency, ω is given by

$$U_{sc}(r, s; \omega) = \int d^3\mathbf{x}\, s^2(\mathbf{x})\, \mathcal{L}_o(\mathbf{x}; \mathbf{r}, \mathbf{s}), \qquad (4)$$

where $\mathcal{L}_o$ is a Fresnel-like volume connecting source to receiver, referred to as a *wavepath* Woodward (1992). The similarity of these two linear integral equations suggests a unified discrete representation

$$\mathbf{d} = \mathbf{A}\, \mathbf{m}, \qquad (5)$$

where $\mathbf{d}$ are the measured data (either traveltime anomalies or waveform anomalies), $\mathbf{m}$ are the model parameters, and the matrix $\mathbf{A}$ is a discrete representation of the integration kernels in either equation (3) or (4).

Inversion of the linear system in equation (5) is carried out by iterative methods, of which the conjugate gradient method is one of the most effective. In the conjugate gradient method repeated iterations of the form

$$\hat{\mathbf{m}}_k = \hat{\mathbf{m}}_{k-1} + \alpha_{k-1}\, \mathbf{A}^T \delta\mathbf{d}$$

are carried out, where α_k is the *step length* at the kth iteration. The quantity $\mathbf{A}^T\delta\mathbf{d}$ has a specific meaning in each of the two different cases:

1. In traveltime tomography this calculates the back*projection* of the traveltime residuals.
2. In waveform tomography this calculates the back*propagation* of the waveform residuals.

Synthetic example

I illustrate a test of the waveform tomography approach in Figure 1: A velocity model containing a random spatial distribution of velocity anomalies was computed numerically (courtesy of Alan Levander and Colin Zelt); synthetic waveform data were generated for this model using a finite difference approach. The survey simulated a crosshole geometry, with 101 sources and 101 receivers. Typical crosshole scale parameters were used: the dominant frequency of the experiment was approximately 300 Hz, leading to a dominant wavelength of approximately 10 m. Wavelengths and the first Fresnel zone width are shown in Figure 1. Traveltimes were picked manually from the waveforms arising in this model, and traveltime tomography using the methods of Zelt and Barton (1998) was used to create a starting velocity model (Figure 1, middle).

The resolution of the traveltime tomogram was somewhat better than the predicted Fresnel zone width, but much worse than the size of the dominant wavelength. Waveform tomography methods were then used to extract a velocity model (starting from the traveltime result) that captured most of the small scale variation of the true model, at better than wavelength scales.

Crosshole examples

Waveform tomography has been successfully applied to crosshole seismic waveforms in a wide variety of geological environments. Examples have been published from scale model experiments (Pratt, 1999), sedimentary environments (Song et al., 1994; Pratt and Shipp, 1999), and most recently from a sub-permafrost investigation of arctic gas hydrates (Pratt et al., 2004). The images in Figure 2 are an example from a site investigation study in crystalline rocks (Albert et al., 1999).

Acknowledgments

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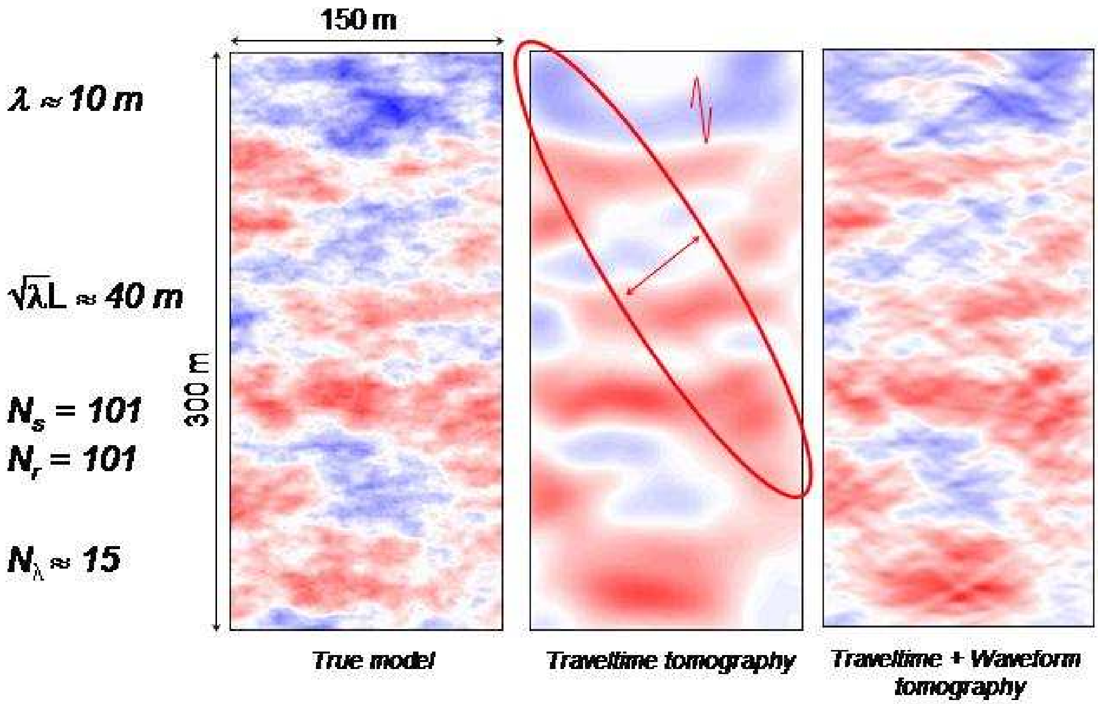
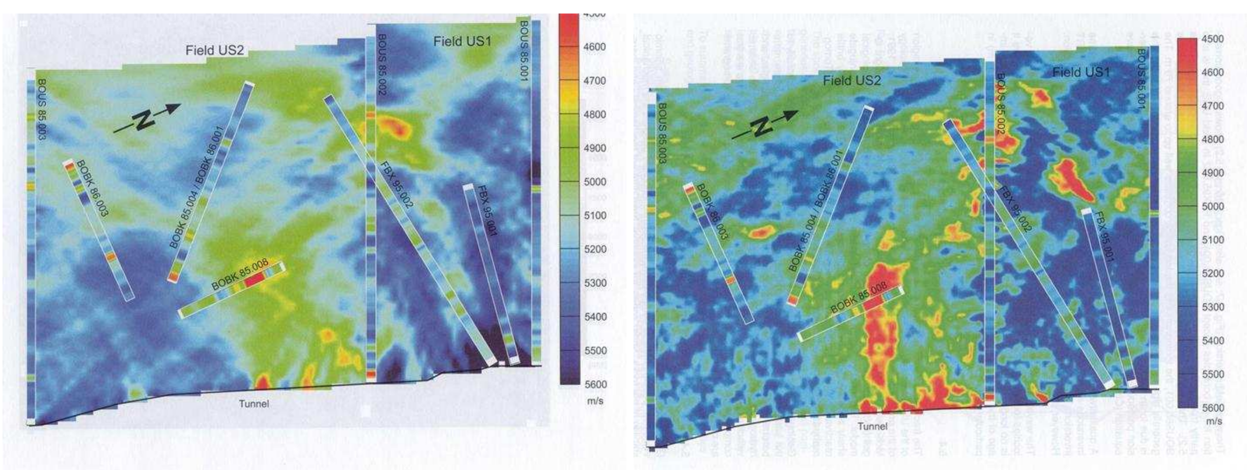


Fig. 1: Results with a random velocity model. Left: the "true" velocity model, middle: the traveltime tomography results and right: the waveform tomography results.



(a) (b)

Fig. 2: Data example from crystalline rock. Crosshole seismic surveys were carried out between three horizontal boreholes in a site investigation survey for the storage of radioactive waste. a) is the traveltime tomography image, and b) is the waveform tomography image.