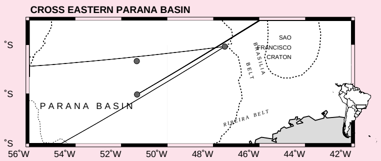
Surface-Wave Inversion for the S-wave Velocity Structure in a Continental Shield Using the Neighbourhood Algorithm

Background

Using linearized least-squares inversion (LLSI), the depth to the top of the low-velocity zone (LVZ) beneath oceans can be found along paths for which surface-wave dispersion is well constrained. The same techniques may give inconclusive results when applied to continental shield regions. The conclusion drawn by Snoke & James (S&J) in their LLSI for the S-wave structure beneath the eastern Paraná Basin in central Brazil is that there is no resolvable LVZ to at least 150 km depth. See Fig. 1 below.



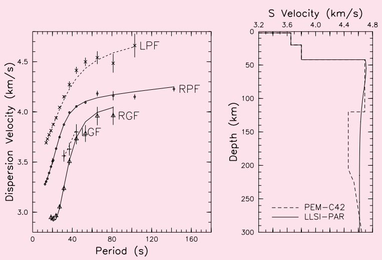


Figure 1: **Top:** Four interstation paths in the Paranß Basin used for surface-wave analysis. (Three are between the same two stations.) **Bottom left:** fundamental-mode Rayleigh and Love Phase and Group velocities. Symbols are the data, and lines are calculated from the best-fit LLSI velocity model. Each dispersion velocity datum is a composite from up to four events, weighted by the inverse of the estimated variances. **Bottom right:** Best-fit S-wave model (LLSI-PAR) calculated using LLSI (solid) compared with the continental PEM model (dashed) for the mantle. The LLSI-PAR velocity model has 43 constant-velocity layers, and the damping used was 0.1 times the maximum eigenvalue of the data kernel matrix.

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References: S&J: JGR, 102, 2939-2951, 1997
NA: GJI, 138, 479-494 and 727--746, 1999 plus http://rses.anu.edu.au/~malcolm/na/

Neighbourhood Algorithm

Because of nonlinearities in the model-data relationships, the damping required to stabilize the inversion and smoothing constraints, the LLSI provides little easily interpreted information about model variance or resolution or the ensemble of acceptable models. Sambridge has introduced the Neighbourhood Algorithm (NA), a direct-search method for nonlinear inversion in a multi-dimensional parameter (model) space, which can be tuned to extract information from an acceptable ensemble of models in addition to finding a single best-fit model. Here we apply NA to the S&J data set to see if it can provide more information about the velocity structure at depth beneath the eastern Parana Basin.

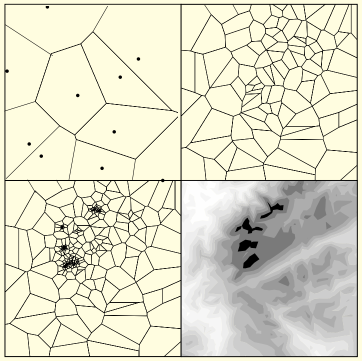

Figure 2: The three stages of a Neighbourhood Algorithm search. The top left panel shows an initial 10 uniformly randomly distributed points and their corresponding Voronoi cells. The top right panel shows the Voronoi cells of the first 100 points generated by the NA, and the bottom left panel shows the Voronoi cells for 500 points. The bottom right panel shows the true fitness landscape, darker shades are higher fitness. With increased sampling in NA, the concentration is much higher in the regions of higher fitness, and all four maxima are found by NA.

NA requires a parameterization of model space, a data set, a method of forward modeling to calculate data for a given model, a definition of the data misfit, three tuning parameters (n_{si}, n_s, and n_r), and N, the number of iterations. The procedure is as follows:

1. **Initialization stage**: First, an initial set of n_{si} models are generated uniformly randomly and a data misfit measure is calculated for each model.
2. **Generation stage**: Next, Voronoi cells (see below) are defined about each of the n_r models with lowest misfit, and a uniform random walk is performed inside each Voronoi cell to generate a total of n_s new models, i.e. n_s/n_r are generated in each cell.
3. **Forward modeling stage**: The data misfit is calculated for the n_s new models generated in step 2 and the procedure returns to step 2. The size and shape of the Voronoi cells automatically adapt to include all previously sampled models.

Steps 2 and 3 constitute an iteration which is then repeated N times resulting in a total of n_{si} + Nn_s model evaluations.. An example (not for this problem) is shown in Fig. 2.

Voronoi cells are nearest neighbour regions as defined by a distance norm. For any set of points (models) in a space with any number of dimensions (unknowns), the Voronoi cells are unique, space filling and non-overlapping. They can be used to partition the model space into a series of neighborhoods.

Procedure

The model parameters used in this study are overlapping, weighted averages over the velocity-depth model (Fig. 3). The data misfit for each model realization is the square of the length of the error vector with each element weighted by the inverse of that datums variance. An additional smoothing constraint is imposed by adding 5.0 to a misfit if successive parameters from among parameters 2 through 8 have opposite signs and differ by more than a preset value (0.175 for these runs). For the two NA runs discussed here, the tuning parameters are n_{si} = 500, n_s = 100, n_r = 50, and N = 95 giving a total of 10,000 model evaluations. Herrmann's SURF routines are used for the forward modeling.

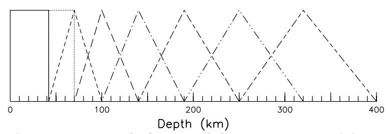

Figure 3: **Interpolation model parameters**: Of the eight parameters, two are box car in shape, resulting in uniform weighting of perturbations to the base-model (the LLSI-PAR model) throughout the depth range, and the remaining six are overlapping triangles. Ranges for these parameters increase from ±0.6 km/s for the crustal velocities (parameter 1) to ±1.75 km/s for parameter 8.

Results

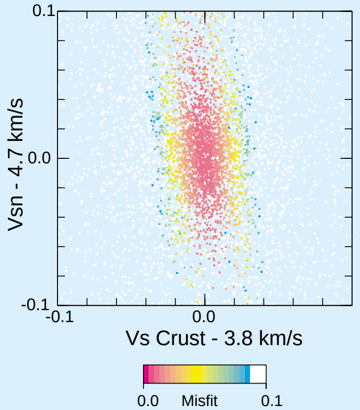

Figure 4: The full ensemble of 10,000 models produced by the NA for the Paranß dispersion data set projected onto a pair of parameter axes. Each model is represented by a dot colour coded by data misfit, and the parameter ranges shown are 1/6 the prescribed hard limits. The concentration of sampling increases in the regions of better data fit, but the full range of values between the hard limits are sampled. The correlations of data fit indicate that the crustal velocity (Vs Crust) is more tightly constrained than the velocity of the uppermost mantle (Vsn).

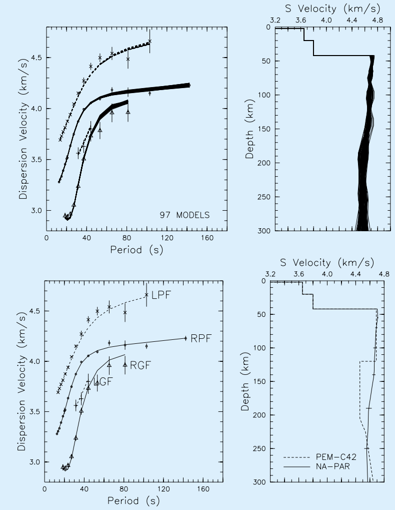

Figure 5: **Top:** Predicted dispersions and models for the ensemble of all models from the NA run total of 10,000 evaluations which have a misfit less than 0.0111. Notation as in Fig. 1. **Bottom:** Average model and its dispersion calculated from the ensemble shown above. The misfit is 0.0107, which would rank it as second among the full ensemble. Calculated (relative) standard deviations for the average velocity model are included.

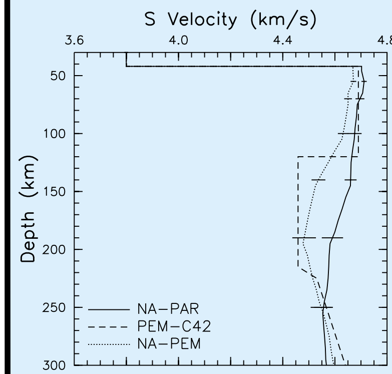

Figure 6: As shown in Fig. 5, the ensemble trend and the average model (labeled NA-PAR) differ significantly from the reference model (labeled PEM-C42). Shown here are those two velocity models along with the model produced from a calibration run (labeled NA-PEM) for which NA was run on a data set with dispersion velocities at the same periods and with the same errors as the data shown in Figs. 1 and 5, but the velocities were fixed to give an exact fit for model PEM-C42. This shows that if there were a LVZ at depths shallower than 150 km, it could be resolved by our data set.

Conclusions

Applying a Neighbourhood Algorithm analysis to dispersion velocity data provides insights not easily seen from LLSI analyses. It is easy to test the relative importance from among the data by doing NA runs with data subsets. By varying the parameterization scheme or the definition of misfit, one can concentrate the analysis on different parts of the model space. For this application, NA confirms the LLSI conclusion that no LVZ begins shallower than 150 km depth, but adds the insight that there is a negative gradient in velocity below 150 km depth. To constrain the velocity structure at greater depths requires dispersion data at higher periods.